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A Comparison of Bootstrap and Monte Carlo Approaches to Testing for Symmetry in the Granger and Lee Error Correction Model

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Abstract: In this paper, I investigate the power of the Granger and Lee model of asymmetry via bootstrap and Monte Carlo techniques. The simulation results indicate that sample size, level of asymmetry and the amount of noise in the data generating process are important determinants of the power of the test for asymmetry based on bootstrap and Monte Carlo techniques. Additionally, the simulation results suggest that both bootstrap and Monte Carlo methods are successful in rejecting the false null hypothesis of symmetric adjustment in large samples with small error size and strong levels of asymmetry. In large samples, with small error size and strong levels of asymmetry, the results suggest that asymmetry test based on Monte Carlo methods achieve greater power gains when compared with the test for asymmetry based on bootstrap. However, in small samples, with large error size and subtle levels of asymmetry, the results suggest that asymmetry test based on bootstrap is more powerful than those based on the Monte Carlo methods. I conclude that both bootstrap and Monte Carlo algorithms provide valuable tools for investigating the power of the test of asymmetry.

Keywords: *Monte Carlo Simulation, Bootstrap Methods, Granger and Lee Model, Power Test, Asymmetry*

1. Introduction

Granger and Lee (1989) propose an approach to modelling asymmetry within the error correction framework. This model allows for asymmetric adjustment by partitioning the Error Correction Term (ECT) about its mean, thus permitting differing speeds of adjustment on either side of the cointegrating vector. However, the failure of the Granger and Lee model to capture asymmetric behaviour in practice led Cook, Holly and Turner (1999a) to examine its power via Monte Carlo simulation. It was found that the Granger and Lee model has low power. Alternatively, Acquah (2012) using bootstrap simulations demonstrated that the Granger and Lee model has low power in rejecting the null of symmetric adjustments in bootstrap samples. A comparison of the bootstrap and Monte Carlo methods is essential since it offers the opportunity to compare and understand the performance of the different simulation methods in the price asymmetry framework. Methodologically, the bootstrap method gives an advantage over the Monte Carlo methods which makes implicit assumptions about the distribution and true values of parameters. The robustness of the bootstrap methods which stems from its lack of reliance on asymptotic theory has not been extensively investigated in the asymmetric price transmission framework. The parametric bootstrap technique is applied in this study against alternative bootstrap methods since the noise variable of the data generating process are identically distributed.

Though Cook, Holly and Turner (1999a) and Acquah (2012) sheds light on the low power of the Granger and Lee model via Monte Carlo and bootstrap simulations respectively, these previous studies fail to provide a comparison of the bootstrap and Monte Carlo methods in the power analysis test. A basic question such as, under what conditions will bootstrap and Monte Carlo techniques of testing for Granger and Lee asymmetry lead to the same results remains unaddressed. Furthermore, an additional question is, under what conditions will the bootstrap method outperform the Monte Carlo methods and vice versa. Empirically, these questions can be addressed in the literature by providing a comparison of Granger and Lee asymmetry test based on bootstrap with those based on Monte Carlo methods. This study is an attempt to fill the gap in the literature. The purpose of this study is to support the claim that the failure of the Granger and Lee model to capture asymmetry in practice is due to low power, and in so doing, provide a comparison of bootstrap and Monte

Carlo based test for asymmetry in the Granger and Lee model. The paper is organized as follows. The introductory section is followed by sections on literature review, testing for symmetry in the Granger and Lee model, Monte Carlo experiments and bootstrap methods. This is followed by the results and discussion and the conclusions of the study.

2. Literature Review

Granger and Lee (1989) were the first to develop a model to test for asymmetric price transmission using an error correction modelling framework. This approach of testing for asymmetry was later modified by various authors (Von Cramon-Taubadel and Loy (1996); Escribano-Pfann (1998) and Tong (1983). The Von Cramon-Taubadel and Loy approach to measuring asymmetry involves specifying asymmetries to affect the direct impact of price increases and decreases as well as the equilibrium relationship. In this model, testing for asymmetry involves the use of a joint F-test. An alternative approach to model asymmetry within the error correction framework is the Escribano-Pfann (EP) model which partitions the ECT using the difference operator. The Escribano-Pfann (EP) approach has been met with success in practice (see Cook, Holly, and Turner, 1999b). Cook, Holly and Turner (2000) demonstrate that the EP model has greater power when compared to the Granger and Lee asymmetric model. Tong (1983) developed the threshold error correction model in which deviation from the long-run equilibrium between the price series will lead to a price response if they exceed a specific threshold level. The Error Correction Term (ECT) is segmented into positive and negative component according to whether it is greater or less than a defined threshold value respectively. In effect, threshold modelling allows for non linear adjustment to equilibrium by introducing the concept of threshold cointegration. Some studies, Cook, Holly and Turner (1999a) and Acquah (2012) have examined the power of the Granger and Lee model via simulations and found that it has low power. However, a comparison of the Granger and Lee asymmetry test based on bootstrap and Monte Carlo approaches has not been extensively examined.

3. Methodology

The methodology describes the Granger and Lee test for asymmetry. Simulation methods employed in the study are presented. Emphasis is given to the parametric bootstrap and Monte Carlo methods.

Testing for Asymmetry in the Granger and Lee Model: Assuming that the true data generating process can be characterized in the following ways:

$$x_t = x_{t-1} + \varepsilon_{1,t} \quad (1)$$

$$\Delta y_t = \beta_1 \Delta x_t - \beta_2^+ (y-x)_{t-1}^+ - \beta_2^- (y-x)_{t-1}^- + \varepsilon_{2,t} \quad (2)$$

$$(y-x)_{t-1}^+ = \begin{cases} (y-x)_{t-1} \; if \; (y-x)_{t-1} > 0 \\ 0 \qquad Otherwise \end{cases} \quad (3)$$

$$(y-x)_{t-1}^- = \begin{cases} (y-x)_{t-1} \; if \; (y-x)_{t-1} < 0 \\ 0 \qquad Otherwise \end{cases} \quad (4)$$

The variable x is assumed to follow a random walk with a normally distributed error term ε_1. The dependent variable y is determined by a potentially asymmetric error correction model with an error term ε_2 which is uncorrelated with the errors driving the x process. There exists an equilibrium relationship between y and x which is defined by the error correction term. Symmetry in equation 2 is detected by determining whether the coefficients (β_2^+ and β_2^-) are identical (that is $H_0 : \beta_2^+ = \beta_2^-$).

Simulation Methods: This study applies both the Monte Carlo and bootstrap simulation techniques. The Monte Carlo experimentation involves drawing the explanatory variable of the regression model and the error term from their respective distribution. If values are assumed for the true model parameters, then the

dependent variable can be obtained and any estimate of interest can be computed. Alternatively, the parametric bootstrap simulation involves sampling from the residual of a parametric model. We begin by estimating the parametric regression model and obtaining the residual. We then resample from the residual to obtain bootstrap samples of the residual. The re-sampled residuals are then added to the explanatory variable to obtain new outcome variable. Using the new outcome variable, the regression is re-estimated and the parameters of interest computed. The outlined process is repeated a large number of times. This process is referred to as parametric bootstrapping. A detailed discussion of the bootstrap methods is presented in Efron and Tibshirani (1993).

4. Results and Discussion

In order to investigate the power of the test for asymmetry under various conditions, a series of bootstrap and Monte Carlo comparison of the Granger and Lee model is carried out based on 10000 replications. In particular, the power of the Granger and Lee model is investigated under conditions of different sample sizes, noise levels and two levels of asymmetry given by $(\beta_2^+, \beta_2^-) \in (0.50, 0.25) or (0.75, 0.25)$. Subtle and strong levels of asymmetry are incorporated into the data generating process. The Granger and Lee model is evaluated in terms of its ability to reject the incorrect null of symmetric adjustment using an F-test of the restricted versus the unrestricted model. The results in Table 1 and 2 indicate the low power of the conventional F-test in rejecting the incorrect null hypothesis of symmetry. Specifically, the Monte Carlo and bootstrap simulations indicate the low power of the conventional F-test in rejecting the null of symmetric adjustment in small sample sizes. For example in small samples with large error size and subtle level of asymmetry, the Monte Carlo method achieved a rejection frequency of 6% compared to 12% for the bootstrap at the 5% significance level as illustrated in the top parts of Tables 1 and 2.

These results are consistent with the Monte Carlo experimentation of Cook, Holly and Turner (1999a). They noted that in small samples with large error size and subtle level of asymmetry, the Monte Carlo method achieved a rejection frequency of 9% at the 5% significance level. Similarly, Acquah (2012) suggests that in small samples with large error size and subtle level of asymmetry, bootstrap method achieved a rejection frequency of 12% at the 5% significance level. This is the same as the current rejection frequency of 12% for bootstrap at the 5% significance level. There is some increase in power when the amount of noise in the data generating process (DGP) is decreased systematically. Noticeably, when the difference in asymmetric adjustment parameters is increased from 0.25 to 0.50 in the true model, an increase in power is also observed in the Granger and Lee test for asymmetry based on bootstrap and Monte Carlo methods as illustrated in Tables 1 and 2. These observations are consistent with Cook, Holly and Turner (1999a) and Acquah (2012) who noted that the power of the test for asymmetry increases with a decrease in noise levels and an increase in the difference in asymmetric adjustment parameters in Monte Carlo and Bootstrap experimentation respectively. However, it is only when the sample size is increased to 500 that a reasonable result is obtained. For example, both the bootstrap and Monte Carlo methods achieve a rejection frequency of approximately 100 % with regards to the rejection of the incorrect null hypothesis of symmetric adjustments at the 5% significance level as illustrated in the bottom parts of Tables 1 and 2. In conclusion, the sample size, difference between the asymmetric adjustment parameters and the amount of noise in the data generating process is important in the power of the test for asymmetry based on bootstrap and Monte Carlo methods. With large sample size or small noise, the Granger and Lee model display greater power in rejecting the (false) null hypothesis of symmetric adjustments.

Table 1: Rejection frequencies based on 10000 Monte Carlo replications

Granger and Lee-Error Correction Model DGP(GL-ECM)				
Sample size	(β_2^+, β_2^-)	**Error Size**	**Rejection Frequencies**	
			5%	**1%**
50	$(-0,25,-0.50)$	3	0.1203	0.0351
50	$(-0,25,-0.50)$	2	0.1230	0.0416
50	$(-0,25,-0.50)$	1	0.1688	0.0687
150	$(-0,25,-0.50)$	3	0.1451	0.0571
150	$(-0,25,-0.50)$	2	0.1857	0.0742
150	$(-0,25,-0.50)$	1	0.3438	0.1912
500	$(-0,25,-0.50)$	3	0.2142	0.1014
500	$(-0,25,-0.50)$	2	0.3212	0.1805
500	$(-0,25,-0.50)$	1	0.7292	0.5614
50	$(-0.25,-0.75)$	3	0.1343	0.0482
50	$(-0.25,-0.75)$	2	0.1739	0.0673
50	$(-0.25,-0.75)$	1	0.3539	0.1893
150	$(-0.25,-0.75)$	3	0.2280	0.1012
150	$(-0.25,-0.75)$	2	0.3490	0.1926
150	$(-0.25,-0.75)$	1	0.7659	0.6048
500	$(-0.25,-0.75)$	3	0.4641	0.2813
500	$(-0.25,-0.75)$	2	0.7302	0.5490
500	$(-0.25,-0.75)$	1	0.9968	0.9875

Table 2: Rejection Frequencies based on 10000 Bootstrap Replications

Granger and Lee-Error Correction Model DGP(GL-ECM)				
Sample size	(β_2^+, β_2^-)	**Error Size**	**Rejection Frequencies**	
			5%	**1%**
50	$(-0,25,-0.50)$	3	0.0551	0.0100
50	$(-0,25,-0.50)$	2	0.0647	0.0142
50	$(-0,25,-0.50)$	1	0.1111	0.0321
150	$(-0,25,-0.50)$	3	0.0796	0.0211
150	$(-0,25,-0.50)$	2	0.1099	0.0323
150	$(-0,25,-0.50)$	1	0.2976	0.1304
500	$(-0,25,-0.50)$	3	0.1493	0.0476
500	$(-0,25,-0.50)$	2	0.2840	0.1188
500	$(-0,25,-0.50)$	1	0.7869	0.5597
50	$(-0.25,-0.75)$	3	0.0756	0.0212
50	$(-0.25,-0.75)$	2	0.1096	0.0322
50	$(-0.25,-0.75)$	1	0.3182	0.1279
150	$(-0.25,-0.75)$	3	0.1580	0.0545
150	$(-0.25,-0.75)$	2	0.3098	0.1285
150	$(-0.25,-0.75)$	1	0.8227	0.6209
500	$(-0.25,-0.75)$	3	0.4461	0.2206
500	$(-0.25,-0.75)$	2	0.7847	0.5784
500	$(-0.25,-0.75)$	1	0.9968	0.9875

5. Conclusion

The power of Granger and Lee approach of detecting asymmetry has been examined using bootstrap and Monte Carlo methods. The results of the bootstrap and Monte Carlo simulations indicate that the power of the Granger and Lee asymmetry depends on various conditions such as sample size, error size and the level of asymmetry. Rejection frequencies of the false null hypothesis of symmetry increases with increase in sample size, increases with increase in difference between the asymmetric adjustment speeds and increases with a decrease in the amount of noise in the true data generating process used in the application. The power of the test for asymmetry based on Monte Carlo and bootstrap methods have rejection frequency of approximately 100% at the 5 percent significance level if the sample size is large with a small error size and strong level of asymmetry. In large samples with small error size and strong level of asymmetry, the test for asymmetry based on Monte Carlo provides greater power. However, in small samples with large error size and subtle level of asymmetry, the test for asymmetry based on bootstrap outperforms the Monte Carlo approach, though both display low power. The low power of the Granger and Lee model in rejecting the null of symmetric adjustment in the Monte Carlo and bootstrap simulations provides an explanation for the failure of the Granger and Lee model to capture asymmetric behaviour in practice. I conclude that both bootstrap and Monte Carlo techniques provide useful algorithms for investigating the power of the test of asymmetry. Furthermore, the study suggests that both the design characteristics and the type of simulation method are important in the power of the test for asymmetry. Future research will extend the current study by testing for the Granger and Lee asymmetry using non parametric bootstrap approach.

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